Mechanics

Chapter 3: WPE (Practice)

Solutions

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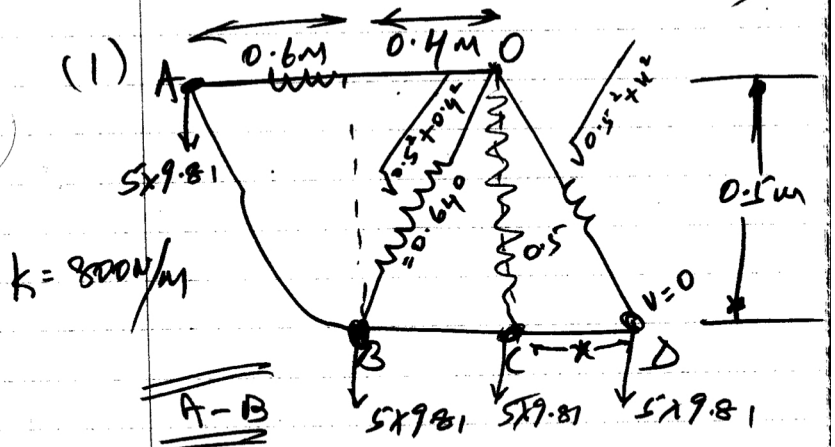
W.P.E. (Practice)

(1)



$$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$$

$$(5\times 9.81)(0.5) + \tfrac{1}{2}\times 8000\left[(1-0.7)^2 + (0.64-0.7)^2\right] = \left(\tfrac{1}{2} m v_B^2\right) - [0]$$

$$\therefore v_B = \frac{\cancel{19.49}}{4.861}\ m/s$$

A-C

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$5 \times 9.81 \times 0.5 + \frac{1}{2} \times 800 \left[(1-0.7)^2 + (0.7-0.5)^2 \right] = \frac{1}{2} M v_C^2 - 0$

$v_C = 4.22 \text{ m/s}$

NOTE: 5×9.81 does work from A-B & not B-C (∵ its ⊥r to displ)

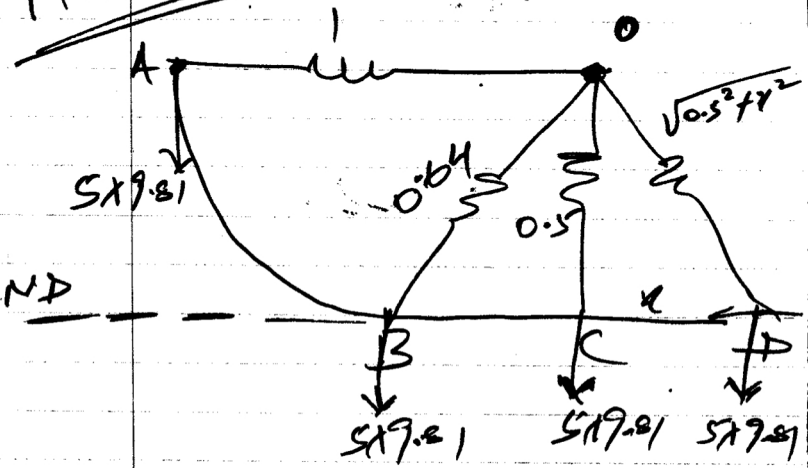
A-D

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$5 \times 9.81 \times 0.5 + \frac{1}{2} \times 800 \left[(1-0.7)^2 + \left(\sqrt{0.5^2 + x^2} - 0.7 \right)^2 \right] = 0 - 0$

$\therefore x = 0.9674 \text{ m}$

Method II



A-B $KE_1 + PE_1 = KE_2 + PE_2$

$(0) + \left[5 \times 9.81 \times 0.5 + \frac{1}{2} \times 800 \times (1-0.7)^2 \right] = \left(\frac{1}{2} \times 5 \times v_B^2 \right) + \left[0 + \frac{1}{2} \times 800 \times (0.7-0.4)^2 \right]$

$V_B = 4.861 \text{ m/s}$

A-C

$\Sigma KE_1 + \Sigma PE_1 = \Sigma KE_2 + \Sigma PE_2$

$(0) + \left[\frac{1}{2} \times 5 \times 9.81 \times 0.5 + \frac{1}{2} \times 800 (1 - 0.7)^2\right]$

$= \left[\frac{1}{2} \times 5 \times v_C^2\right] + \left[0 + \frac{1}{2} \times 800 \times (0.7 - 0.5)^2\right]$

$\therefore v_C = 4.22 \text{ m/s}$

A-D $\Sigma KE_1 + \Sigma PE_1 = \Sigma KE_2 + \Sigma PE_2$

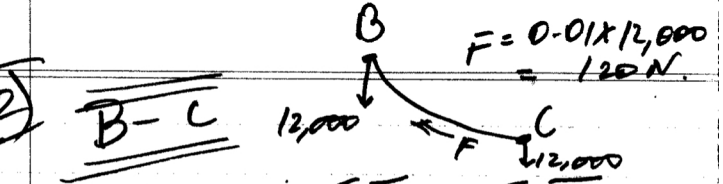
$(0) + \left[\frac{1}{2} \times 5 \times 9.81 \times 0.5 + \frac{1}{2} \times 800 (1 - 0.7)^2\right]$

$= \left[\frac{1}{2} \times 5 \times v_D^2\right] + \left[0 + \frac{1}{2} \times 800 (\sqrt{0.5^2 + x^2} - 0.7)^2\right]$

$x = 0.967 \text{ m}$

P.2

B-C



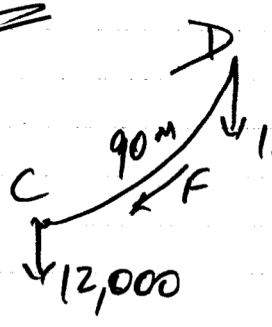
$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$(12,000 \times 6) - 120(50) = \frac{1}{2} \times \frac{12000}{9.81} \times (12 \times \frac{5}{18})^2 - \frac{1}{2} \times \frac{12000}{9.81} (v_B)^2$

$\therefore v_B = 17.091 \text{ m/s}$

C-D

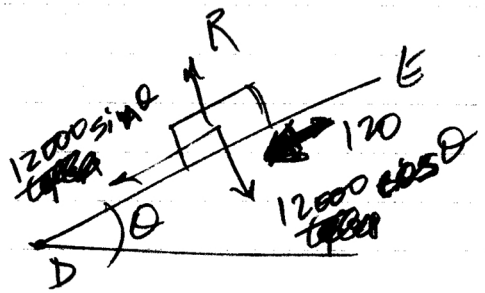
$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$



$12,000 \times (\quad) - 120(90) = \frac{1}{2} \times \frac{12000}{9.81} \times v_D^2 - \frac{1}{2} \times \frac{12000}{9.81} \times (\quad)^2$

$v_D = 15.013 \text{ m/s}$

D-E



$\tan\theta = \frac{1}{80}$

$\theta = 0.716°$

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$-12000\sin\theta(x) - 120(x)$

$= \frac{1}{2} \times \frac{12000}{9.81}(0)^2 - \frac{1}{2} \times \frac{12000}{9.81}(v_D)^2$

$\therefore x = 510.65 \text{ m}$

$v^2 = u^2 - 2as$

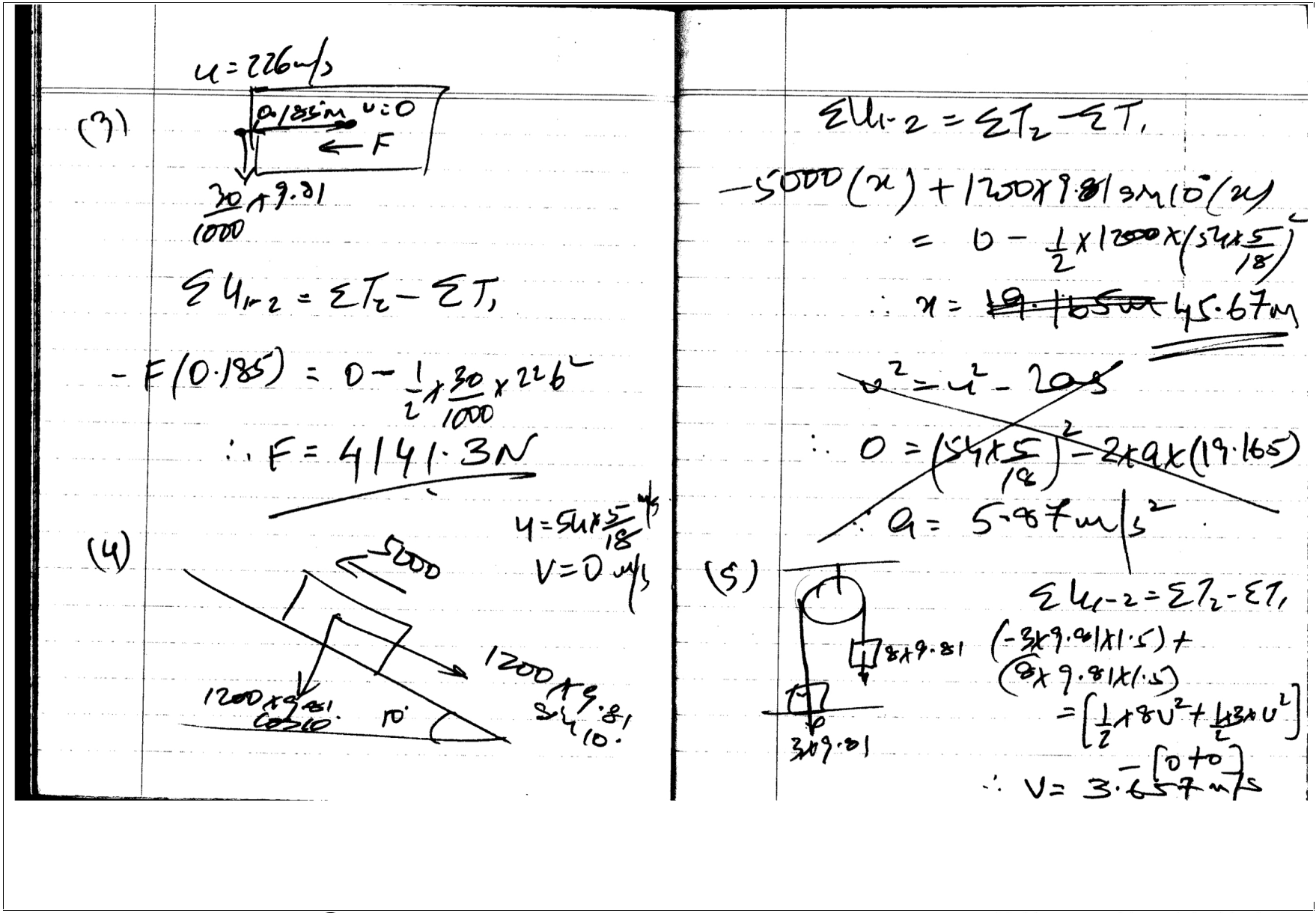
$0 = 15.013^2 - 2(a)(510.65)$

$\therefore a = 0.221 \text{ m/s}^2$

$v = u - at$

$0 = 15.013 - 0.221(t)$

$\therefore t = 68.02 \text{ s}$



(3)

$u = 22.6\ m/s$

$0.185\ m$ $v = 0$

$\leftarrow F$

$\frac{30}{1000} \times 9.81$

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$-F(0.185) = 0 - \frac{1}{2} \times \frac{30}{1000} \times 22.6^2$

$\therefore F = 4141.3\ N$

(4)

$u = 54 \times \frac{5}{18}\ m/s$

$v = 0\ m/s$

5000

$1200 \times 9.81 \cos 10°$

$1200 \times 9.81 \sin 10°$

$10°$

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$-5000(x) + 1200 \times 9.81 \sin 10°(x)$

$= 0 - \frac{1}{2} \times 1200 \times (54 \times \frac{5}{18})^2$

$\therefore x = 45.67\ m$

$v^2 = u^2 - 2as$

$\therefore 0 = (54 \times \frac{5}{18})^2 - 2 \times a \times (19.165)$

$\therefore a = 5.87\ m/s^2$

(5)

8×9.81

3×9.81

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

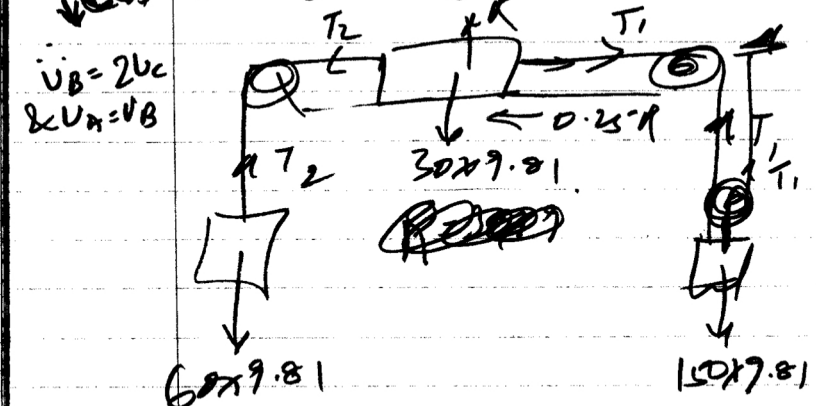
$(-3 \times 9.81 \times 1.5) + (8 \times 9.81 \times 1.5) = [\frac{1}{2} \times 8v^2 + \frac{1}{2} \times 3v^2] - [0 + 0]$

$\therefore v = 3.657\ m/s$

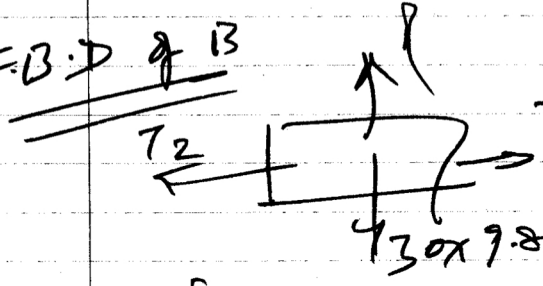
NOTE: If C goes up by 1.5m
B & A " down " 3m

$\therefore v_B = 2v_C$ & $v_A = v_B$

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$



F.B.D of B



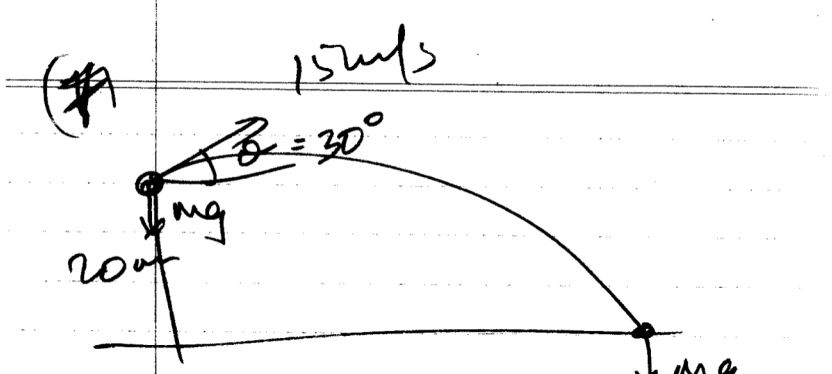
$\Sigma F_y = 0$ $\therefore R = 30 \times 9.81$

Combined System

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$150 \times 9.81 \times 1.5 - 0.25R(+3) - 60 \times 9.81(3)$
$= \left(\frac{1}{2} \times 150 \times v^2 + \frac{1}{2} \times 30 \times (2v)^2 + \frac{1}{2} \times 60 \times (2v)^2\right) - [0 + 0 + 0]$

$\therefore v_C = 0.93 \text{ m/s}$



$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

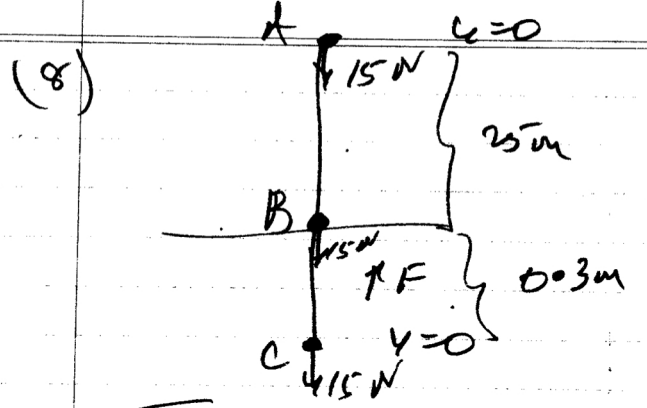
$mg(20) = \frac{1}{2}mv^2 - \frac{1}{2}m(15)^2$

$\therefore v = 24.85 \text{ m/s}$

Method 2: $\Sigma KE_1 + \Sigma PE_1 = \Sigma KE_2 + \Sigma PE_2$

$\left(\frac{1}{2}m(15)^2\right) + mg(20) = \left(\frac{1}{2}mv^2\right) + (0)$

$\therefore v = 24.85 \text{ m/s}$



A-C

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$15 \times (25 + 0.3) - F(0.3) = 0 \rightarrow$

$\therefore F = 1265 \text{ N}$

$v^2 = u^2 - 2gs$

$v^2 = 0 - 2 \times 9.81 \times (-25)$ Thus, $v = \underline{22.147}$ m/s

B-C

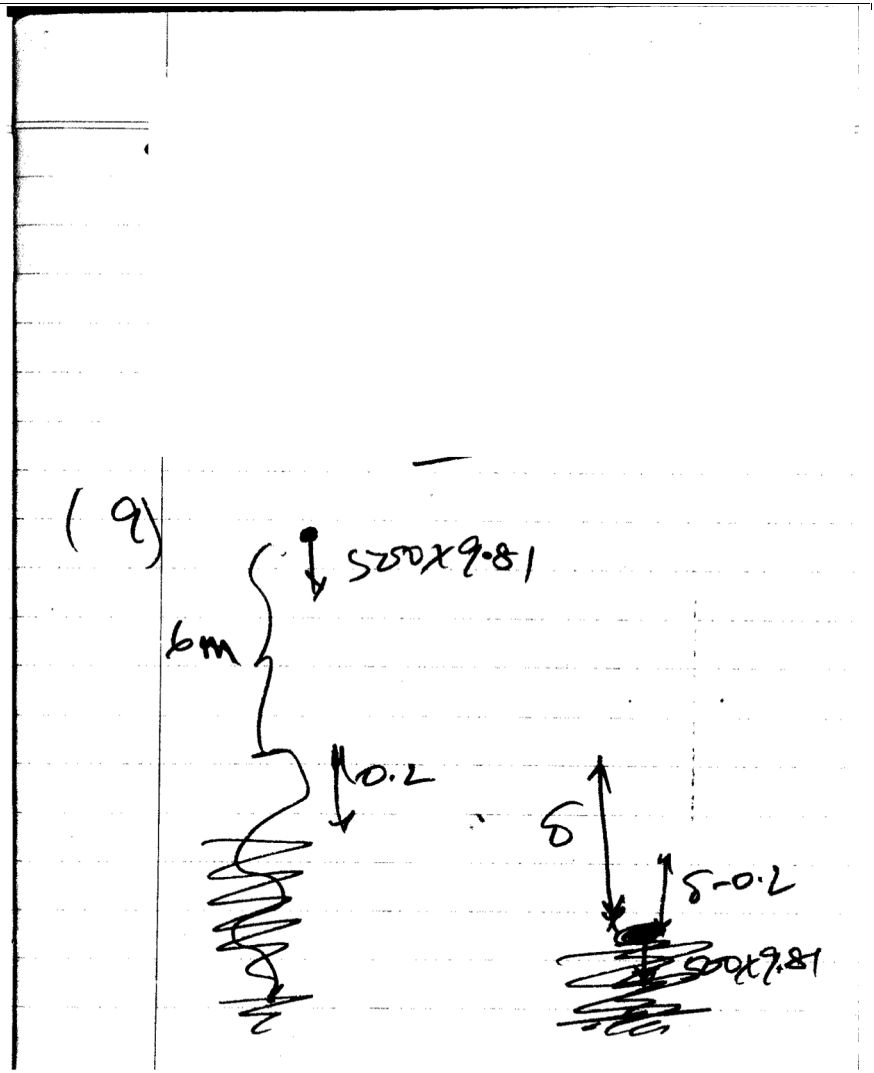
$v^2 = u^2 - 2as$

$0 = 22.147^2 - 2 \times a \times (0.3)$

$a = 817.483 \text{ m/s}^2$

$v = u - at$

$0 = 22.147 - (817.483) \times t$
$t = 0.027s$

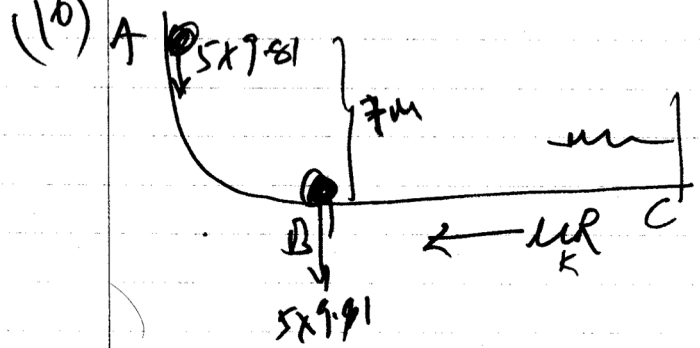


$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$500 \times 9.81 \times (6+\delta) + 4 \times \frac{1}{2} \times 10{,}000 (0^2 - \delta^2)$
$+ 4 \times \frac{1}{2} \times 15{,}000 [0^2 - (\delta - 0.2)^2] = 0 - 0$

$\therefore \delta = 0.9392 \text{ m}$

(10)



~~(10)~~ A-B

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$5 \times 9.81 \times 7 = \frac{1}{2} M v^2 - 0$

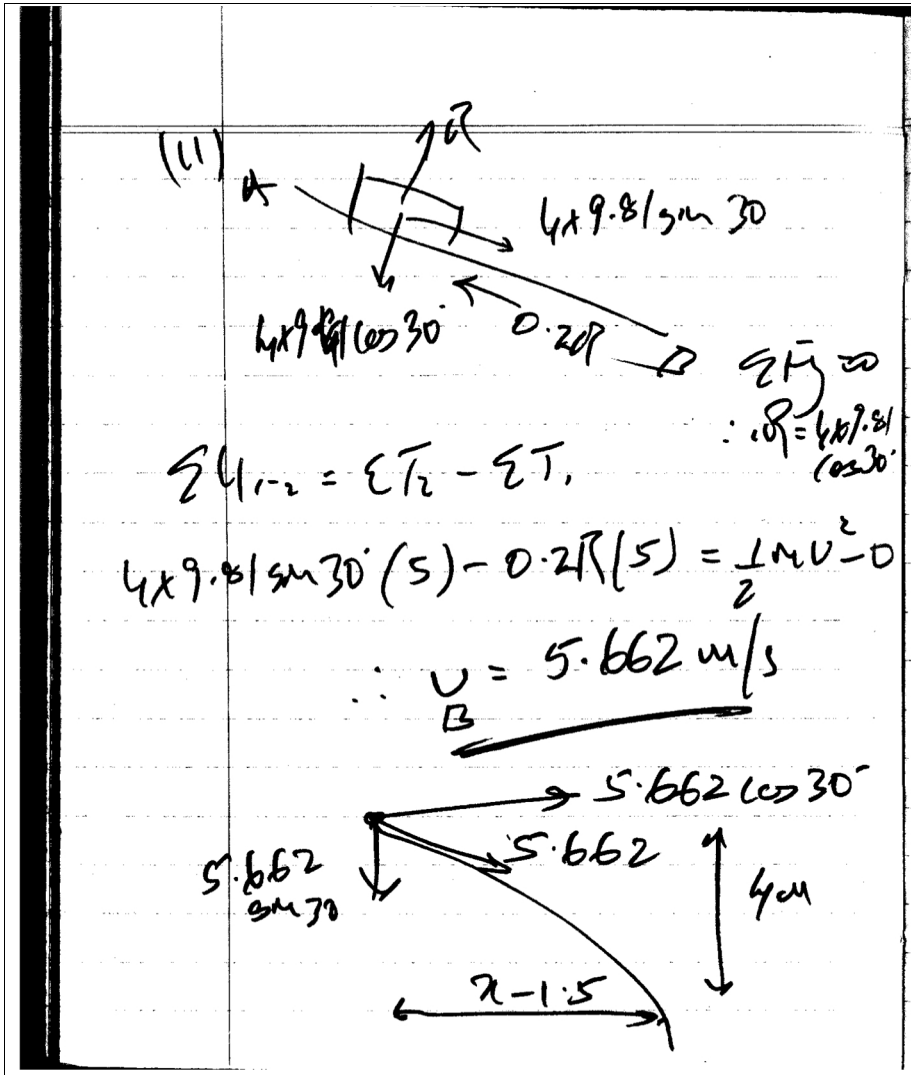
$\therefore v_B = 11.719 \text{ m/s}$

A-C

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$5 \times 9.81 \times 7 - 0.2(R)(10+\delta)$
$+ \frac{1}{2} \times 900 (0^2 - \delta^2) = 0 - 0$

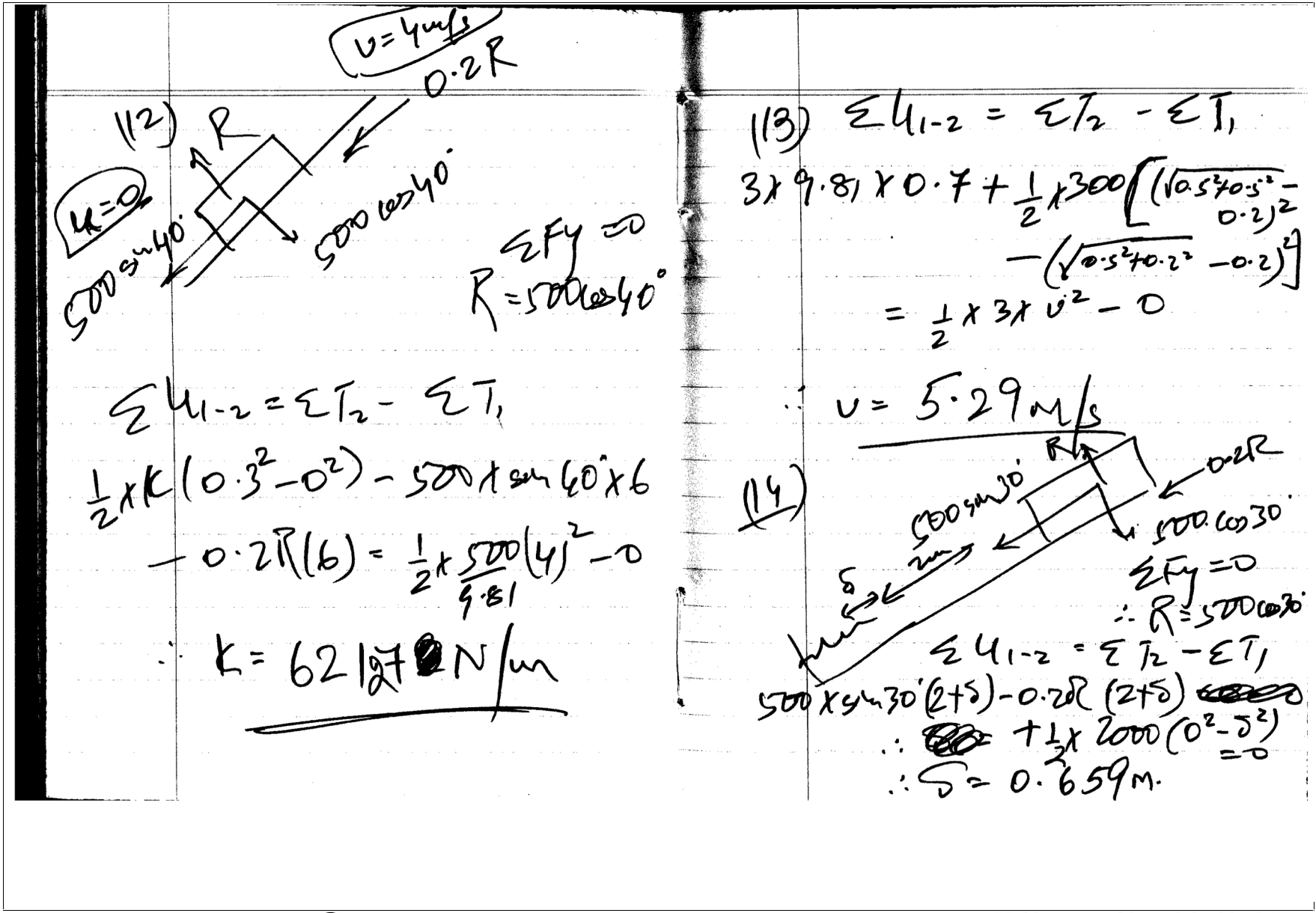
$\therefore \delta = 0.727 \text{ m}.$



Horizontal	Vertical
$s = x - 1.5$	$s = -4\,m$
$4 = 5.662 \cos 30°$	$4 = -5.662 \sin 30°$
$s = ut$	$s = ut - \frac{1}{2}gt^2$
$\therefore t = \frac{x - 1.5}{5.662 \cos 30°}$	

$$-4 = (-5.662 \sin 30°)\left(\frac{x-1.5}{5.662 \cos 30°}\right) - \frac{1}{2}\times 9.81\left(\frac{x-1.5}{5.662 \cos 30°}\right)^2$$

$$\therefore x = 4.734\,m$$



(12) $v = 4\ m/s$

$0.2R$

R

$K = 0$

$500\sin 40°$

$500\cos 40°$

$\Sigma F_y = 0$

$R = 500\cos 40°$

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$\frac{1}{2} \times K(0.3^2 - 0^2) - 500 \times \sin 40° \times 6 - 0.2R(6) = \frac{1}{2} \times \frac{500}{9.81}(4)^2 - 0$

$\therefore K = 6212.7\ N/m$

(13) $\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$3 \times 9.81 \times 0.7 + \frac{1}{2} \times 300\left[\left(\sqrt{0.5^2 + 0.5^2} - 0.2\right)^2 - \left(\sqrt{0.5^2 + 0.2^2} - 0.2\right)^2\right] = \frac{1}{2} \times 3 \times v^2 - 0$

$\therefore v = 5.29\ m/s$

(14)

R

$0.2R$

$500\sin 30°$

$500\cos 30°$

$2m$

S

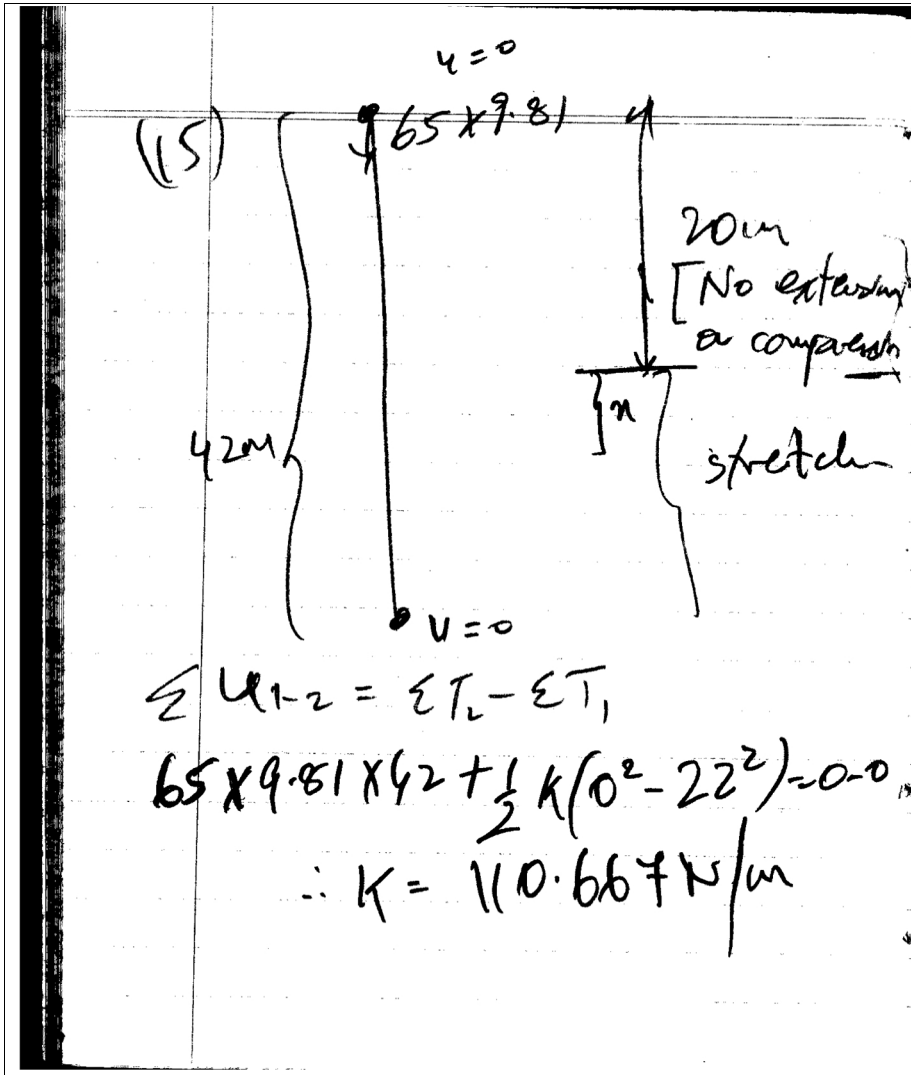
$\Sigma F_y = 0$

$\therefore R = 500\cos 30°$

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$500 \times \sin 30°(2+S) - 0.2R(2+S) + \frac{1}{2} \times 2000(0^2 - 5^2) = 0$

$\therefore S = 0.659\ m.$



(15)

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$65 \times 9.81 \times 42 + \frac{1}{2} k(0^2 - 22^2) = 0 - 0$

$\therefore k = 110.667\ N/m$

Let dist be x when max. velocity is obtained.

$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$

$65 \times 9.81 (20 + x) + \frac{1}{2} \times k(0^2 - x^2) = \frac{1}{2} \times 65 \times v_{max}^2 - 0 \Rightarrow (1)$

Taking derivative w.r.t x

$65 \times 9.81 + \frac{1}{2} k(-2x) = 65 \frac{dv}{dx}$

Put $dv/dx = 0$

$\therefore x = 5.762m$

ie $25.762m$ from top of body

Sub in (1)

$v_{max} = 21.18\ m/s$

(16) $V = 20,000$ litres

$= \frac{20000}{1000}\ m^3$

density $= 1000\ kg/m^3$

$m = \rho \times V$

$\therefore m = 20,000\ kg$

$t = 30 \times 60 = 1800\ s$

$s = 15\ m$.

Energy $= mgh$

$= 20,000 \times 9.8 \times 15$

Power $= \frac{\text{Energy}}{\text{Time}}$

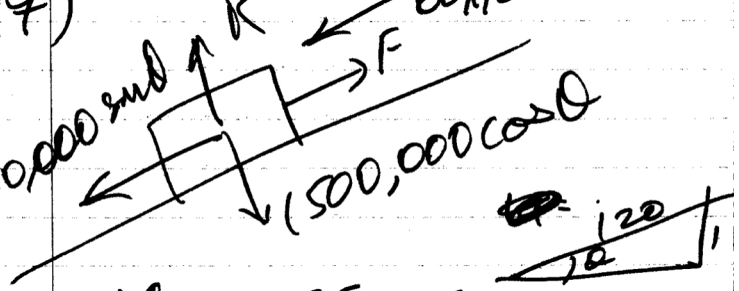
$= 1633\ W$

Eff. $= \frac{\text{Power o/p}}{\text{Power input}}$

$\therefore$ Power input $= \frac{1633}{0.6}$

$= 2725\ W$

(17)



$\Sigma E_{1-2} = \Sigma T_2 - \Sigma T_1$

$-1500,000 \sin\theta (1.5\times10^3) - 60\times1500\times(1.5\times10^3) + F\times1.5\times10^3 = \frac{1}{2}m\left(\frac{36\times5}{18}\right)^2 - \frac{1}{2}m\left(\frac{36\times5}{18}\right)^2$

$F = 102500\ N$

$\text{Work} = Fs$

$= 102500 \times (1.5 \times 10^3)$

$= 153.75 \times 10^6\ J$

$s = ut + \frac{1}{2}at^2$

$a = 0 \quad (\because u = v)$

$\therefore s = ut$

$1500 = \left(36 \times \frac{5}{18}\right) t$

$\therefore t = 150\ s.$

$\therefore \text{Power} = \frac{\text{Work}}{\text{Time}}$

$= \frac{153.75 \times 10^6}{150}$

$= 1025\ kW$

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